Joint Spatial and Angular Representation for Sparse Reconstruction of HARDI

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Diffusion magnetic resonance imaging (dMRI) measures random motion of water molecules along fibers in brain tissue [Descoteaux 08]. A common technique used to acquire dMRI images is called high angular resolution diffusion imaging (HARDI). HARDI measures diffusion by sampling along N gradient directions on the sphere ($\mathbb{S}^2$) [Merlet 09]. The HARDI signal S is a 6D signal. In order to visualize this signal we think about diffusion weighted images (DWI). Each DWI provides information about water diffusion in a brain volume for a given gradient direction. Furthermore, for each gradient direction, we are directing a magnet in a specific gradient direction, which allows us to measure water diffusion along that direction.HARDI data can be conveniently represented using a single mathematical function (known as orientation distribution functions, or ODFs for short) that not only visualizes the location of fibers but also the directions in which fibers are aligned in brain tissue [Descoteaux 08].

Advanced dMRI techniques such as HARDI require hundreds of diffusion weighted images to estimate accurate probability distributions of water diffusion in the brain requiring extended periods of human scanning time, which is deemed clinically infeasible. Methods to reduce the number of diffusion measurements is needed to make these advanced neuroimaging protocols clinically feasible.

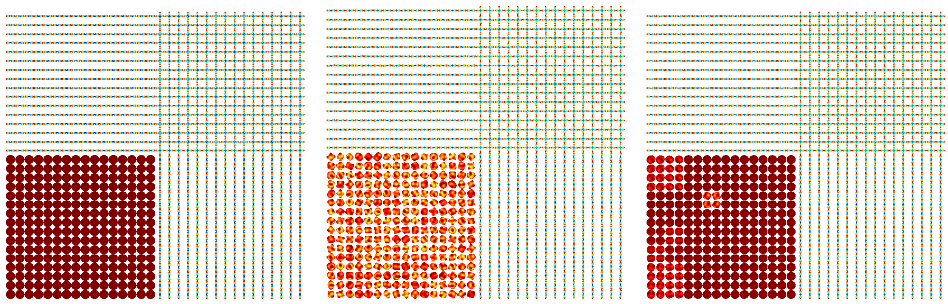
Most state-of-the-art compressed sensing methods for HARDI construct or learn basis elements to sparsely represent a diffusion signal or probability distribution in each voxel separately with regulatory terms to capture similarities between neighboring voxels [Merlet, Goh]. However, neighboring voxels with similar signals will then be modeled with redundant sets of basis elements and redundant sets of sparse coefficients in each voxel. These approaches for estimating ODFs model the spatial and angular domains of HARDI data *separately.*

We assert that by constructing or learning bases to represent patches of voxels instead of the same basis for each voxel, we will be able to achieve a much sparser code for reconstructing diffusion signals. By modeling patches of voxels with a single basis we exploit redundancies of neighboring voxels with a single set of sparse coefficients. For this project, we model the spatial and angular domains of HARDI data *jointly.* We accomplish this by utilizing two powerful mathematical tools: real-valued symmetric spherical harmonic (SH) basis functions and 2D Haar wavelet basis functions. The SH basis is used to represent the angular domain by modeling spherical functions, whereas the wavelet basis models 2D images.

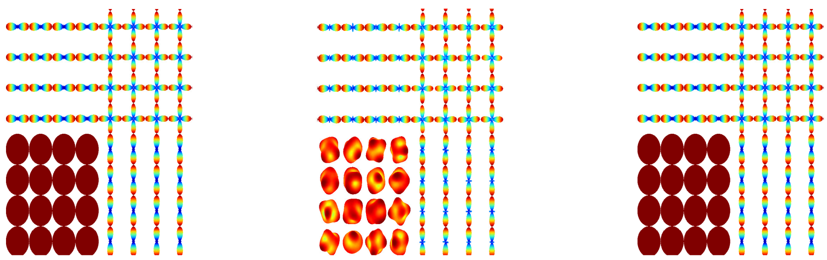
Our signal of interest is a function of two variables: x, the location of a voxel in $\mathbb{R}^3$ and $\vec{g}$, the sample gradient direction on the unit sphere $\mathbb{S}^2$. For a given voxel x and gradient direction $\vec{g}$, $S(x,\vec{g})$ can be expressed using a joint basis in which $\{Y_i\}$ are SH basis functions and $\psi_j(x)$ are wavelet basis functions:

$$S(x, \vec{g}) \quad = \quad \sum_{i}^{M}\sum_{j}^{W} c'_{i,j}\psi_j(x)Y_i(\vec{g}) \tag{1}$$

In our experiments, we estimate the ODFs from reconstruction of the noise-corrupted datasets. Below are the results shown for a synthetic dataset that models the crossings of horizontal and vertical fibers.



(a) Actual ODFs



(b) Close ups of ODFs

Figure 1: (Left) Ground Truth ODF (SNR = ∞). (Middle) Corrupted ODF (SNR = 30). (Right) ODF from 2D HAAR reconstruction (Sparse Level = 20, Tol = 15%)

We found that the joint Haar wavelet & SH reconstructions were successful for varying levels of noise. We attribute this to the Haar wavelet's block structure, which is exploited in HARDI datasets that have sharp edges between differing orientation distribution functions. In exploring the joint sparsity of our HARDI datasets, we suspect that the 2D Haar wavelet basis would not accurately represent the spatial domain of realistic datasets. In analyzing synthetic tissue phantom data it perhaps would be more convenient to explore other wavelets such as the 2D Daubechies wavelet. In addition to this, we need to adopt a model that fully represents the spatial domain of our realistic HARDI datasets as three dimensional spaces. On the other hand Orthogonal Matching Pursuit (OMP) might help obtain efficient HARDI reconstructions.

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